Trigtools Package

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We use open-source computer algebra system(CAS) maxima 5.31.2. The trigtools package contains commands that help you work with trigonometric expessions. List of functions in trigtools package:

c2sin c2cos

The function c2sin convert expression a*cos(x)+b*sin(x) to r*sin(x+phi).

The function c2cos convert expression a*cos(x)+b*sin(x) to r*cos(x-phi).

Examples:

```
(%i1) load(trigtools)$
```

```
(%i2) c2sin(3*sin(x)+4*cos(x));
```

$$(\%o2)\quad 5\sin\left(x+\operatorname{atan}\left(\frac{4}{3}\right)\right)$$

```
(%i3) trigexpand(%),expand;
```

$$(\%o3)\quad 3\sin(x)+4\cos(x)$$

```
(%i4) c2cos(3*sin(x)-4*cos(x));
```

$$(\%o4)\quad -5\cos\left(x+\operatorname{atan}\left(\frac{3}{4}\right)\right)$$

```
(%i5) trigexpand(%),expand;
```

$$(\%o5)\quad 3\sin(x)-4\cos(x)$$

```
(%i6) c2sin(sin(x)+cos(x));
```

$$(\%o6)\quad \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$$

```
(%i7) trigexpand(%),expand;
```

$$(\%o7)\quad \sin(x)+\cos(x)$$

```
(%i8) c2cos(sin(x)+cos(x));
```

$$(\%o8)\quad \sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$$

```
(%i9) trigexpand(%),expand;
```

$$(\%o9)\quad \sin(x)+\cos(x)$$

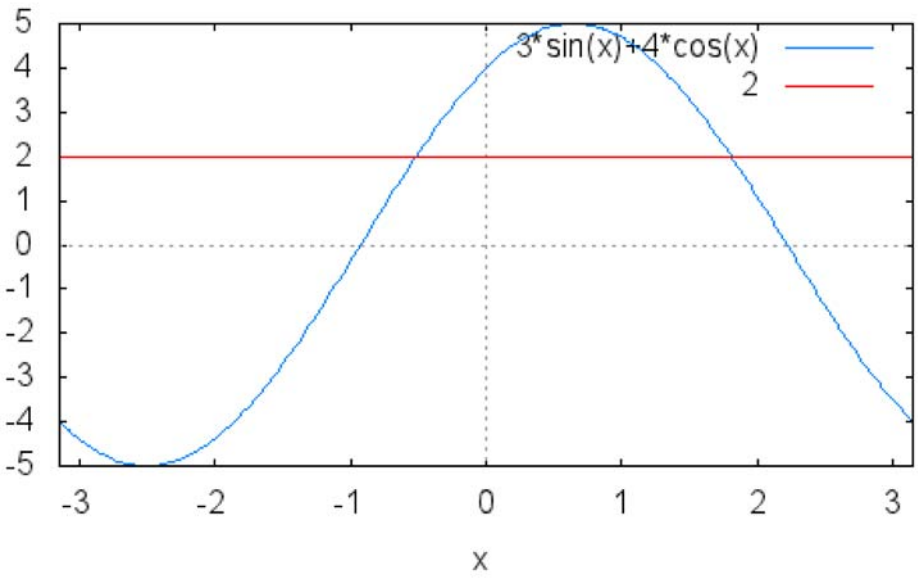
Example. Solve trigonometric equation

```
(%i10) eq:3*sin(x)+4*cos(x)=2;
```

$$(\%o10)\quad 3\sin(x)+4\cos(x)=2$$

```
(%i11) wxplot2d([3*sin(x)+4*cos(x),2],[x,-%pi,%pi]);
```

(%t11)



(%o11)

```
(%i12) eq1:c2sin(lhs(eq))=2;
```

(%o12) $5\sin\left(x+\operatorname{atan}\left(\frac{4}{3}\right)\right)=2$

```
(%i13) solvetrigwarn:false$
```

```
(%i14) solve(eq1)[1]$ x1:rhs(%);
```

(%o15) $\operatorname{asin}\left(\frac{2}{5}\right)-\operatorname{atan}\left(\frac{4}{3}\right)$

```
(%i16) float(%), numer;
(%o16) -0.51577837193412
```

```
(%i17) eq2:c2cos(lhs(eq))=2;
```

(%o17) $5\cos\left(x-\operatorname{atan}\left(\frac{3}{4}\right)\right)=2$

```
(%i18) solve(eq2,x)[1]$ x2:rhs(%);
```

(%o19) $\operatorname{atan}\left(\frac{3}{4}\right)+\operatorname{acos}\left(\frac{2}{5}\right)$

```
(%i20) float(%), numer;
(%o20) 1.802780589520693
```

```
(%i21) sol:[x1,x2];
```

(%o21) $\left[\operatorname{asin}\left(\frac{2}{5}\right)-\operatorname{atan}\left(\frac{4}{3}\right),\operatorname{atan}\left(\frac{3}{4}\right)+\operatorname{acos}\left(\frac{2}{5}\right)\right]$

Answ.: x = x1 + 2*pi*k, x1 = asin(2/5)-atan(4/3) or x1 = atan(3/4)+acos(2/5), k -- any integer.

c2trig

The function c2trig (convert to trigonometric) reduce expression with hyperbolic functions sinh, cosh, tanh, coth to trigonometric expression with sin, cos, tan, cot.

```
(%i1) load(trigtools)$
```

Examples:

1.

```
(%i2) sinh(x)=c2trig(sinh(x));
      cosh(x)=c2trig(cosh(x));
      tanh(x)=c2trig(tanh(x));
      coth(x)=c2trig(coth(x));
(%o2) sinh(x)=-%i sin(%i x)
(%o3) cosh(x)=cos(%i x)
(%o4) tanh(x)=-%i tan(%i x)
(%o5) coth(x)=%i cot(%i x)
```

2. see http://www.math.utexas.edu/pipermail/maxima/2013/034585.html

```
(%i6) cos(p+q*%i);
(%o6) cos(%i q+p)

(%i7) trigexpand(%);
(%o7) cos(p)cosh(q)-%i sin(p)sinh(q)

(%i8) c2trig(%);
(%o8) cos(%i q+p)
```

3.

```
(%i9) sin(a+b*%i);
(%o9) sin(%i b+a)

(%i10) trigexpand(%);
(%o10) %i cos(a)sinh(b)+sin(a)cosh(b)

(%i11) c2trig(%);
(%o11) sin(%i b+a)
```

4.

```
(%i12) cos(a*%i+b*%i);
(%o12) cos(%i b+%i a)
```

```
(%i13) trigexpand(%);
(%o13) sinh(a)sinh(b)+cosh(a)cosh(b)

(%i14) c2trig(%);
(%o14) cos(%i b+%i a)
```

5.

```
(%i15) tan(a+%i*b);
(%o15) tan(%i b+a)

(%i16) trigexpand(%);
                    %i tanh(b)+tan(a)
(%o16)              ------------------
                    1-%i tan(a)tanh(b)

(%i17) c2trig(%);
(%o17) tan(%i b+a)
```

6.

```
(%i18) cot(x+%i*y);
(%o18) cot(%i y+x)

(%i19) trigexpand(%);
                    -%i cot(x)coth(y)-1
(%o19)              -------------------
                      cot(x)-%i coth(y)

(%i20) c2trig(%);
(%o20) cot(%i y+x)
```

c2hyp

The function c2h (convert to hyperbolic) convert expression with exp function to expression with hyperbolic functions sinh, cosh.

```
(%i5) load(trigtools)$
```

Examples:

```
(%i6) c2hyp(exp(x));
(%o6) sinh(x)+cosh(x)
```

```
(%i7) c2hyp(exp(x)+exp(x^2)+1);
(%o7) sinh(x^2)+cosh(x^2)+sinh(x)+cosh(x)+1
```

```
(%i8) c2hyp(exp(x)/(2*exp(y)-3*exp(z)));
```

$$(\%o8)\quad \frac{\sinh(x)+\cosh(x)}{2(\sinh(y)+\cosh(y))-3(\sinh(z)+\cosh(z))}$$

trigfactor

The function trigfactor factors expresions of form +-sin(x)+-cos(y)

```
(%i1) load(trigtools)$
```

Examples:

1.

```
(%i2) trigfactor(sin(x)+cos(x));
```

$$(\%o2)\ \sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$$

```
(%i3) trigrat(%);
```

$$(\%o3)\ \sin(x)+\cos(x)$$

2.

```
(%i4) trigfactor(sin(x)+cos(y));
```

$$(\%o4)\ 2\cos\left(\frac{y}{2}-\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{y}{2}+\frac{x}{2}-\frac{\pi}{4}\right)$$

```
(%i5) trigrat(%);
```

$$(\%o5)\ \cos(y)+\sin(x)$$

3.

```
(%i6) trigfactor(sin(x)-cos(3*y));
```

$$(\%o6)\ 2\sin\left(\frac{3y}{2}-\frac{x}{2}+\frac{\pi}{4}\right)\sin\left(\frac{3y}{2}+\frac{x}{2}-\frac{\pi}{4}\right)$$

```
(%i7) trigrat(%);
```

$$(\%o7)\ \sin(x)-\cos(3y)$$

4.

```
(%i8) trigfactor(-sin(5*x)-cos(3*y));
```

$$(\%o8)\ -2\cos\left(\frac{3y}{2}-\frac{5x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{3y}{2}+\frac{5x}{2}-\frac{\pi}{4}\right)$$

```
(%i9) trigrat(%);
```

$$(\%o9)\ -\cos(3y)-\sin(5x)$$

5.

```
(%i10) sin(alpha)+sin(beta)=trigfactor(sin(alpha)+sin(beta));
```

$$\sin(\beta)+\sin(\alpha)=2\cos\left(\frac{\beta}{2}-\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}+\frac{\alpha}{2}\right)$$

```
(%i11) trigrat(%);
```

$$\sin(\beta)+\sin(\alpha)=\sin(\beta)+\sin(\alpha)$$

6.

```
(%i12) sin(alpha)-sin(beta)=trigfactor(sin(alpha)-sin(beta));
```

$$\sin(\alpha)-\sin(\beta)=-2\sin\left(\frac{\beta}{2}-\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}+\frac{\alpha}{2}\right)$$

7.

```
(%i13) cos(alpha)+cos(beta)=trigfactor(cos(alpha)+cos(beta));
```

$$\cos(\beta)+\cos(\alpha)=2\cos\left(\frac{\beta}{2}-\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}+\frac{\alpha}{2}\right)$$

8.

```
(%i14) cos(alpha)-cos(beta)=trigfactor(cos(alpha)-cos(beta));
```

$$\cos(\alpha)-\cos(\beta)=2\sin\left(\frac{\beta}{2}-\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}+\frac{\alpha}{2}\right)$$

9

```
(%i15) trigfactor(3*sin(x)+7*cos(x));
```

$$3\sin(x)+7\cos(x)$$

```
(%i16) c2sin(%);
```

$$\sqrt{58}\sin\left(x+\operatorname{atan}\left(\frac{7}{3}\right)\right)$$

```
(%i17) trigexpand(%),expand;
```

$$3\sin(x)+7\cos(x)$$

10.

```
(%i18) trigfactor(sin(2*x));
```

$$\sin(2x)$$

```
(%i19) trigexpand(%);
```

$$2\cos(x)\sin(x)$$

trigsolve

The function trigsolve find solutions of trigonometric equation from interval [a, b).

```
(%i1) load(trigtools)$
```

Examples:

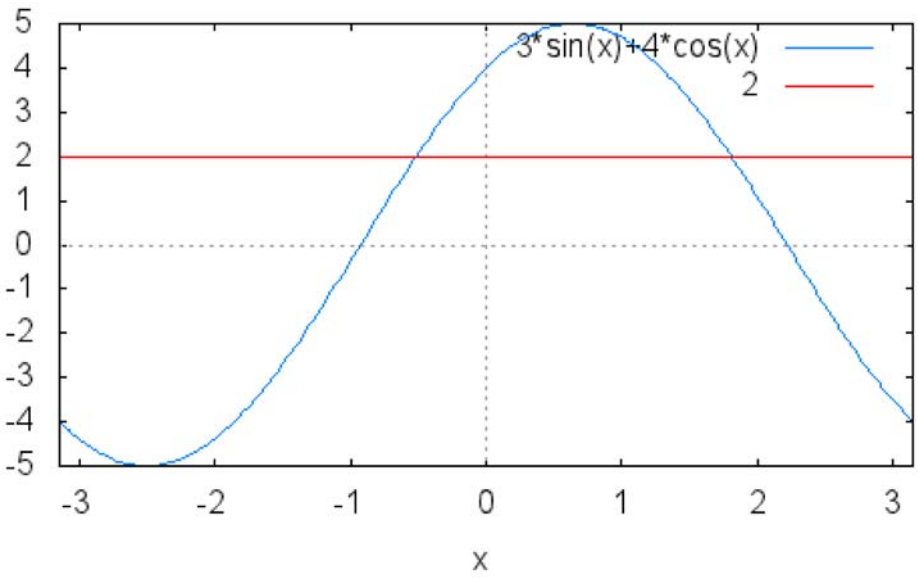
1.

```
(%i38) eq:eq:3*sin(x)+4*cos(x)=2;
```

$$(\%o38)\quad 3\sin(x)+4\cos(x)=2$$

```
(%i39) wxplot2d([3*sin(x)+4*cos(x),2],[x,-%pi,%pi]);
```

(%t39)



(%o39)

```
(%i40) sol:trigsolve(eq,-%pi,%pi);
```

$$(\%o40)\quad \left\{\operatorname{atan}\left(\frac{2\sqrt{21}}{5}-\frac{12}{5}\right),\ \pi-\operatorname{atan}\left(\frac{2\sqrt{21}}{5}+\frac{12}{5}\right)\right\}$$

```
(%i41) float(%), numer;
(%o41) {-0.51577837193412,1.802780589520693}
```

Answ. : x = atan((2*sqrt(21))/5-12/5)+2*%pi*k; x=%pi-atan((2*sqrt(21))/5+12/5)+2*%pi*k, k -- any integer.

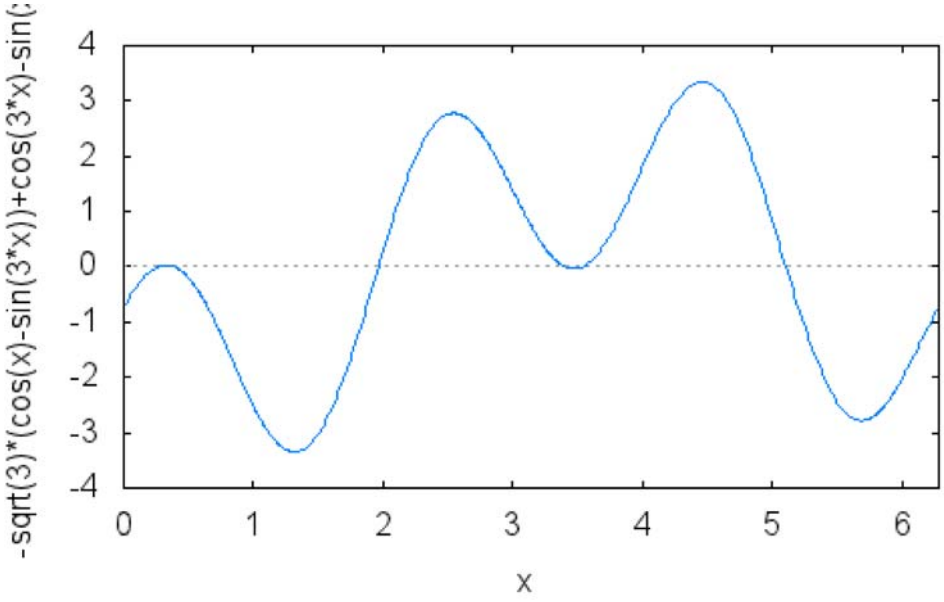
2.

```
(%i6) eq:cos(3*x)-sin(x)=sqrt(3)*(cos(x)-sin(3*x));
```

$$(\%o6)\quad \cos(3x)-\sin(x)=\sqrt{3}\,(\cos(x)-\sin(3x))$$

```
(%i7) wxplot2d([lhs(eq)-rhs(eq)], [x,0,2*%pi])$
```

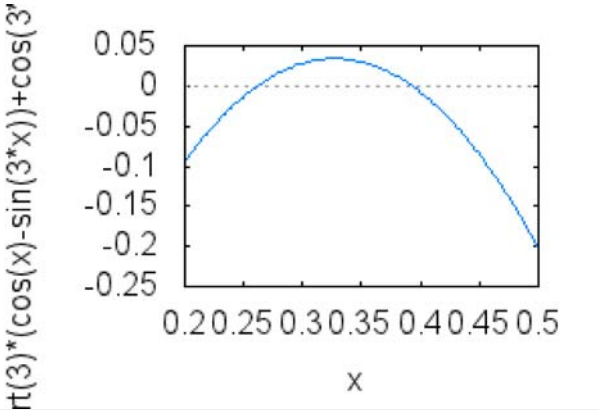
(%t7)



We have 6 solutions from [0, 2*pi].

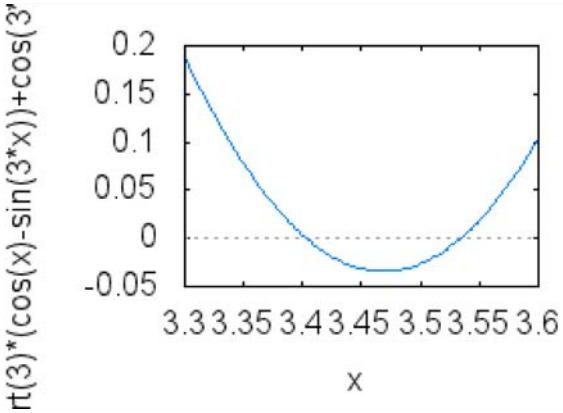
```
(%i8) wxplot2d([lhs(eq)-rhs(eq)], [x,0.2,0.5]),wxplot_size=[300,200]$
```

(%t8)




```
(%i9) wxplot2d([lhs(eq)-rhs(eq)], [x,3.3,3.6]),wxplot_size=[300,200]$
```

(%t9)




```
(%i10) trigfactor(lhs(eq))=map(trigfactor,rhs(eq));
```

(%o10) $$-2\sin\left(x+\frac{\pi}{4}\right)\sin\left(2x-\frac{\pi}{4}\right)=2\sqrt{3}\sin\left(x-\frac{\pi}{4}\right)\sin\left(2x-\frac{\pi}{4}\right)$$

```
(%i11) factor(lhs(%)-rhs(%));
```

$$(\%o11)\ -2\left(\sin\left(\frac{4x+\pi}{4}\right)+\sqrt{3}\sin\left(\frac{4x-\pi}{4}\right)\right)\sin\left(\frac{8x-\pi}{4}\right)$$

Equation is equivalent to

```
(%i12) L:factor(rhs(%)-lhs(%));
```

$$(\%o12)\ 2\left(\sin\left(\frac{4x+\pi}{4}\right)+\sqrt{3}\sin\left(\frac{4x-\pi}{4}\right)\right)\sin\left(\frac{8x-\pi}{4}\right)$$

```
(%i13) eq1:part(L,2)=0;
```

$$(\%o13)\ \sin\left(\frac{4x+\pi}{4}\right)+\sqrt{3}\sin\left(\frac{4x-\pi}{4}\right)=0$$

```
(%i14) eq2:part(L,3)=0;
```

$$(\%o14)\ \sin\left(\frac{8x-\pi}{4}\right)=0$$

```
(%i15) S1:trigsolve(eq1,0,2*%pi);
```

$$(\%o15)\ \left\{\frac{\pi}{12},\frac{13\pi}{12}\right\}$$

```
(%i16) S2:trigsolve(eq2,0,2*%pi);
```

$$(\%o16)\ \left\{\frac{\pi}{8},\frac{5\pi}{8},\frac{9\pi}{8},\frac{13\pi}{8}\right\}$$

```
(%i17) S:listify(union(S1,S2));
```

$$(\%o17)\ \left[\frac{\pi}{12},\frac{\pi}{8},\frac{5\pi}{8},\frac{13\pi}{12},\frac{9\pi}{8},\frac{13\pi}{8}\right]$$

```
(%i18) float(%), numer;
(%o18) [0.26179938779915,0.39269908169872,1.963495408493621,
3.403392041388942,3.534291735288517,5.105088062083414]
```

Answer:
x = a+2*%pi*k, where a any from S, k any integer

3.

```
(%i19) eq:8*cos(x)*cos(4*x)*cos(5*x)-1=0;
```

$$(\%o19)\ 8\cos(x)\cos(4x)\cos(5x)-1=0$$

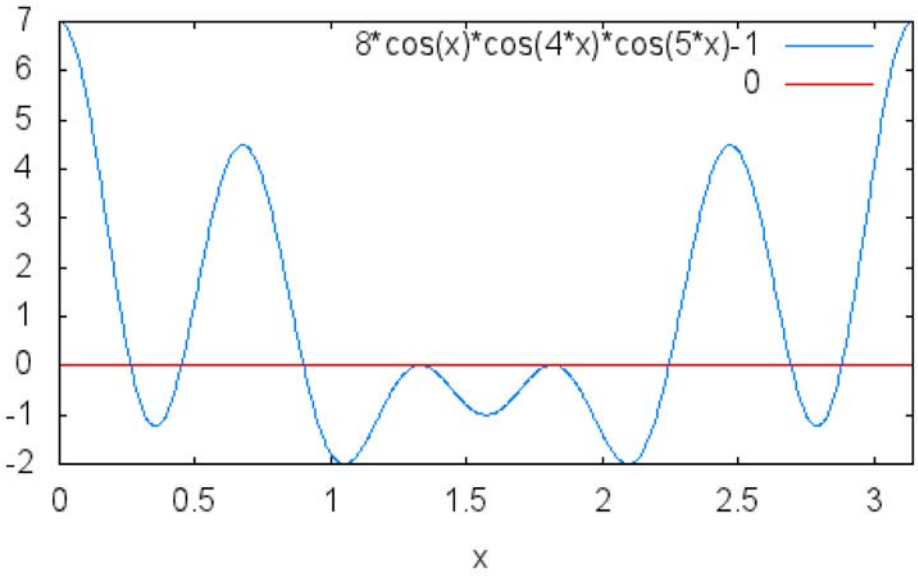
```
(%i20) trigrat(%);
```

$$(\%o20)\ 2\cos(10x)+2\cos(8x)+2\cos(2x)+1=0$$

Left side is periodic with period T=pi.

```
(%i21) wxplot2d([lhs(eq),rhs(eq)],[x,0,%pi]);
```

(%t21)



(%o21)

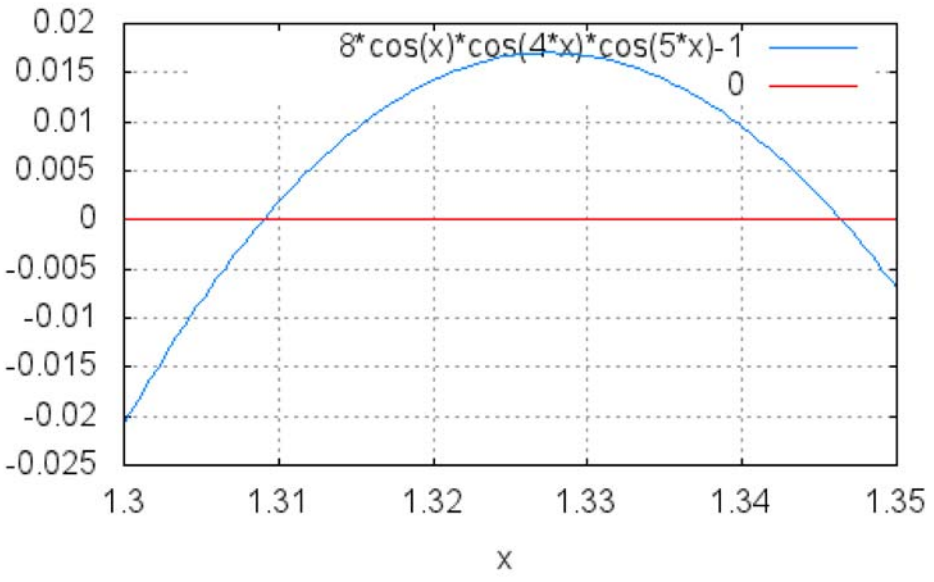
We have 10 solutions from [0, pi].

```
(%i22) x4:find_root(eq, x, 1.3, 1.32);
(%o22) 1.308996938995747
```

```
(%i23) x5:find_root(eq, x, 1.32, 1.35);
(%o23) 1.346396851538483
```

```
(%i24) wxplot2d([lhs(eq),0],[x,1.3,1.35],
       [gnuplot_preamble, "set grid;"]
       );
```

(%t24)



(%o24)

Equation we multiplty by 2*sin(x)*cos(2*x) :

```
(%i25) eq*2*sin(x)*cos(2*x);
```
$$(\%o25)\quad 2\sin(x)\cos(2x)(8\cos(x)\cos(4x)\cos(5x)-1)=0$$

```
(%i26) eq1:trigreduce(%),expand;
```
$$(\%o26)\quad \sin(13x)+\sin(x)=0$$

```
(%i27) trigfactor(lhs(eq1))=0;
```
$$(\%o27)\quad 2\cos(6x)\sin(7x)=0$$

```
(%i28) S1:trigsolve(cos(6*x),0,%pi);
```
$$(\%o28)\quad \left\{\frac{\pi}{12},\frac{\pi}{4},\frac{5\pi}{12},\frac{7\pi}{12},\frac{3\pi}{4},\frac{11\pi}{12}\right\}$$

```
(%i29) S2:trigsolve(sin(7*x),0,%pi);
```
$$(\%o29)\quad \left\{0,\frac{\pi}{7},\frac{2\pi}{7},\frac{3\pi}{7},\frac{4\pi}{7},\frac{5\pi}{7},\frac{6\pi}{7}\right\}$$

We remove solutions of sin(x)=0 and cos(2x)=0:

```
(%i30) S3:trigsolve(sin(x),0,%pi);
(%o30) {0}
```

```
(%i31) S4:trigsolve(cos(2*x),0,%pi);
```
$$(\%o31)\quad \left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$$

We find 10 solutions from [0, pi] :

```
(%i32) union(S1,S2)$ setdifference(%,S3)$ setdifference(%,S4);
```
$$(\%o34)\quad \left\{\frac{\pi}{12},\frac{\pi}{7},\frac{2\pi}{7},\frac{5\pi}{12},\frac{3\pi}{7},\frac{4\pi}{7},\frac{7\pi}{12},\frac{5\pi}{7},\frac{6\pi}{7},\frac{11\pi}{12}\right\}$$

```
(%i35) S:listify(%);
```
$$(\%o35)\quad \left[\frac{\pi}{12},\frac{\pi}{7},\frac{2\pi}{7},\frac{5\pi}{12},\frac{3\pi}{7},\frac{4\pi}{7},\frac{7\pi}{12},\frac{5\pi}{7},\frac{6\pi}{7},\frac{11\pi}{12}\right]$$

```
(%i36) length(S);
(%o36) 10
```

```
(%i37) float(S), numer;
(%o37) [0.26179938779915,0.44879895051283,0.89759790102566,
1.308996938995747,1.346396851538483,1.79519580205131,
1.832595714594046,2.243994752564138,2.692793703076966,
2.879793265790644]
```

Answer:
x = a+2*%pi*k, where a any from S, k any integer

trigvalue, trigeval

The function trigvalue compute values of sin(m*pi/n), cos(m*pi/n), tan(m*pi/n), cot(m*pi/n) in radicals.

The function trigeval compute values of expressions with sin(m*pi/n), cos(m*pi/n), tan(m*pi/n), cot(m*pi/n) in radicals.

Examples:

1 Values of trigonometric functions

```
(%i1) load(trigtools)$
```

```
(%i2) trigvalue(sin(%pi/10));
```

$$(\%o2)\ \frac{\sqrt{5}-1}{4}$$

```
(%i3) trigvalue(cos(%pi/10));
```

$$(\%o3)\ \frac{\sqrt{\sqrt{5}+5}}{2^{3/2}}$$

```
(%i4) trigvalue(tan(%pi/10));
```

$$(\%o4)\ \frac{\sqrt{5-2\sqrt{5}}}{\sqrt{5}}$$

```
(%i5) float(%), numer;
(%o5) 0.32491969623291
```

```
(%i6) float(tan(%pi/10)), numer;
(%o6) 0.32491969623291
```

```
(%i7) trigvalue(cot(%pi/10));
```

$$(\%o7)\ \sqrt{2\sqrt{5}+5}$$

```
(%i8) float(%), numer;
(%o8) 3.077683537175254
```

```
(%i9) float(cot(%pi/10)), numer;
(%o9) 3.077683537175254
```

```
(%i10) trigvalue(sin(%pi/32));
```

$$(\%o10)\ \frac{\sqrt{2-\sqrt{\sqrt{\sqrt{2}+2}+2}}}{2}$$

(%i11) `trigvalue(cos(%pi/32));`

(%o11) $$\frac{\sqrt{\sqrt{\sqrt{\sqrt{2}+2}+2}+2}}{2}$$

(%i12) `trigvalue(cos(%pi/256));`

(%o12) $$\frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}+2}+2}+2}+2}+2}+2}}{2}$$

(%i13) `trigvalue(cos(%pi/60));`

(%o13) $$\frac{\sqrt{\sqrt{\sqrt{2}\,\sqrt{3}\,\sqrt{\sqrt{5}+5}+\sqrt{5}+7}+4}}{2^{3/2}}$$

(%i14) `trigvalue(sin(%pi/60));`

(%o14) $$\frac{\sqrt{4-\sqrt{\sqrt{2}\,\sqrt{3}\,\sqrt{\sqrt{5}+5}+\sqrt{5}+7}}}{2^{3/2}}$$

(%i15) `trigvalue(sin(%pi/18));`

(%o15) $$\sin\left(\frac{\pi}{18}\right)$$

(%i16) `trigvalue(sin(%pi/20));`

(%o16) $$\frac{\sqrt{4-\sqrt{2}\,\sqrt{\sqrt{5}+5}}}{2^{3/2}}$$

2 ode example

(%i17) `load(odes)$`

(%i18) `eq:'diff(y,x,5)+2*y=0;`

(%o18) $$\frac{d^5}{d\,x^5}y+2\,y=0$$

(%i19) `odeL(eq,y,x);`

(%o19) $$y=\%e^{-2^{1/5}\cos\left(\frac{4\,\pi}{5}\right)x}\sin\left(2^{1/5}\sin\left(\frac{4\,\pi}{5}\right)x\right)C5+\%e^{-2^{1/5}\cos\left(\frac{4\,\pi}{5}\right)x}\cos\left(2^{1/5}\sin\left(\frac{4\,\pi}{5}\right)x\right)C4+\%e^{-2^{1/5}\cos\left(\frac{2\,\pi}{5}\right)x}\sin\left(2^{1/5}\sin\left(\frac{2\,\pi}{5}\right)x\right)C3+\%e^{-2^{1/5}\cos\left(\frac{2\,\pi}{5}\right)x}\cos\left(2^{1/5}\sin\left(\frac{2\,\pi}{5}\right)x\right)C2+\%e^{-2^{1/5}\,x}\,C1$$

```
(%i20) sol:trigeval(%);
```

$$(\%o20)\quad y=\%e^{\frac{(\sqrt{5}+1)x}{2^{9/5}}}\sin\left(\frac{\sqrt{5-\sqrt{5}}\,x}{2^{13/10}}\right)C5+\%e^{\frac{(\sqrt{5}+1)x}{2^{9/5}}}\cos\left(\frac{\sqrt{5-\sqrt{5}}\,x}{2^{13/10}}\right)C4+\%e^{-\frac{(\sqrt{5}-1)x}{2^{9/5}}}\sin\left(\frac{\sqrt{\sqrt{5}+5}\,x}{2^{13/10}}\right)C3+\%e^{-\frac{(\sqrt{5}-1)x}{2^{9/5}}}\cos\left(\frac{\sqrt{\sqrt{5}+5}\,x}{2^{13/10}}\right)C2+\%e^{-2^{1/5}x}C1$$

Test:

```
(%i21) subst(sol,eq)$
```

```
(%i22) ev(%, nouns)$
```

```
(%i23) radcan(%);
```

$$(\%o23)\quad 0=0$$

3 n-th root of complex number

Example. Find the 4-th roots of %i

```
(%i24) solve(x^4=%i,x);
```

$$(\%o24)\quad [x=(-1)^{1/8}\,\%i,\ x=-(-1)^{1/8},\ x=-(-1)^{1/8}\,\%i,\ x=(-1)^{1/8}]$$

```
(%i25) rectform(%);
```

$$(\%o25)\quad [x=\%i\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right),\ x=-\%i\sin\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{8}\right),\ x=\sin\left(\frac{\pi}{8}\right)-\%i\cos\left(\frac{\pi}{8}\right),\ x=\%i\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)]$$

```
(%i26) trigeval(%);
```

$$(\%o26)\quad [x=\frac{\sqrt{\sqrt{2}+2}\,\%i}{2}-\frac{\sqrt{2-\sqrt{2}}}{2},\ x=-\frac{\sqrt{2-\sqrt{2}}\,\%i}{2}-\frac{\sqrt{\sqrt{2}+2}}{2},\ x=\frac{\sqrt{2-\sqrt{2}}}{2}-\frac{\sqrt{\sqrt{2}+2}\,\%i}{2},\ x=\frac{\sqrt{2-\sqrt{2}}\,\%i}{2}+\frac{\sqrt{\sqrt{2}+2}}{2}]$$

atan_contract

The function atan_contract(r) contracts atan functions. We assume: abs(r)<pi/2.

Examples:

```
(%i1) load(trigtools)$
```

1.

```
(%i2) atan_contract(atan(x)+atan(y));
```

$$(\%o2)\quad \operatorname{atan}(y)+\operatorname{atan}(x)$$

```
(%i3) assume(abs(atan(x)+atan(y))<%pi/2)$
```

```
(%i4) atan(x)+atan(y)=atan_contract(atan(x)+atan(y));
```

$$(\%o4)\quad \operatorname{atan}(y)+\operatorname{atan}(x)=\operatorname{atan}\left(\frac{y+x}{1-x\,y}\right)$$

2.

```
(%i5) atan(1/3)+atan(1/5)+atan(1/7)+atan(1/8)$ %=atan_contract(%);
```

$$(\%o6)\quad \operatorname{atan}\left(\frac{1}{3}\right)+\operatorname{atan}\left(\frac{1}{5}\right)+\operatorname{atan}\left(\frac{1}{7}\right)+\operatorname{atan}\left(\frac{1}{8}\right)=\frac{\pi}{4}$$

3.

Machin's formulae

```
(%i7) 4*atan(1/5)-atan(1/239)=atan_contract(4*atan(1/5)-atan(1/239));
```

$$(\%o7)\quad 4\operatorname{atan}\left(\frac{1}{5}\right)-\operatorname{atan}\left(\frac{1}{239}\right)=\frac{\pi}{4}$$

4.

see http://en.wikipedia.org/wiki/Machin-like_formula

```
(%i8) 12*atan(1/49)+32*atan(1/57)-5*atan(1/239)+12*atan(1/110443)$
      %=atan_contract(%);
```

$$(\%o9)\quad 12\operatorname{atan}\left(\frac{1}{49}\right)+32\operatorname{atan}\left(\frac{1}{57}\right)-5\operatorname{atan}\left(\frac{1}{239}\right)+12\operatorname{atan}\left(\frac{1}{110443}\right)=\frac{\pi}{4}$$

References:

1. http://maxima.sourceforge.net/