ODES Package

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We use open-source computer algebra system(CAS) maxima 5.31.2.
The ODES package contains commands that help you work with ordinary differential equations. List of functions in ODES package:

odecv

Function: odecv(tr,eq,y,x)
Makes the change of independed variable in ODE.

```
(%i1) load(odes)$
```

Examples:

1. x^3y''' + xy' - y = x

```
(%i2) eq:x^3*'diff(y,x,3)+x*'diff(y,x)-y=x$
```

```
(%i3) odecv(x=exp(t),eq,y,x);
```

(%o3) $\frac{d^3}{d\, t^3} y - 3 \left(\frac{d^2}{d\, t^2} y \right) + 3 \left(\frac{d}{d\, t} y \right) - y = \%e^t$

```
(%i4) odeL(%,y,t);
```

(%o4) $y = t^2\, \%e^t\, C3 + t\, \%e^t\, C2 + \%e^t\, C1 + \frac{t^3\, \%e^t}{6}$

```
(%i5) sol:subst(t=log(x),%);
```

(%o5) $y = x\, \log(x)^2\, C3 + x\, \log(x)\, C2 + x\, C1 + \frac{x\, \log(x)^3}{6}$

2. (1 + x^2)y'' + xy' + y = 0

```
(%i6) eq:(1+x^2)*'diff(y,x,2)+x*'diff(y,x)+y=0$
```

```
(%i7) trans(eq):=block(
      coeff(lhs(eq),'diff(y,x))/coeff(lhs(eq),'diff(y,x,2)),
      t=radcan(integrate(exp(-integrate(%%,x)),x)))$
```

```
(%i8) itr:trans(eq); tr:solve(itr,x)[1];
```

(%o8) $t = \operatorname{asinh}(x)$

(%o9) $x = \sinh(t)$

```
(%i10) odecv(tr,eq,y,x);
```

(%o10) $\frac{d^2}{d\, t^2} y + y = 0$

```
(%i11) ode2(%,y,t);
```

(%o11) $y = \%k1\, \sin(t) + \%k2\, \cos(t)$

```
(%i12) sol:subst(itr,%);
```

(%o12) $y = \%k1\, \sin(\operatorname{asinh}(x)) + \%k2\, \cos(\operatorname{asinh}(x))$

3. y'' - y'/x + 4x^2y = 0

```
(%i13) eq:'diff(y,x,2)-1/x*'diff(y,x)+4*x^2*y=0$
(%i14) itr:t=x^2; tr:x=sqrt(t);
```

(%o14) $t=x^2$

(%o15) $x=\sqrt{t}$

```
(%i16) odecv(t=x^2,eq,y,x);
```

(%o16) $4\,t\left(\frac{d^2}{d\,t^2}\,y\right)+4\,t\,y=0$

```
(%i17) ode2(%,y,t);
```

(%o17) $y=\%k1\sin(t)+\%k2\cos(t)$

```
(%i18) sol:subst(itr,%);
```

(%o18) $y=\%k1\sin(x^2)+\%k2\cos(x^2)$

4. kamke 3.74. 4x^4 y'''-4x^3 y'' + 4x^2 y' = 1

```
(%i19) eq:4*x^4*'diff(y,x,3)-4*x^3*'diff(y,x,2)+4*x^2*'diff(y,x)=1$
(%i20) odecv(x=exp(t),eq,y,x);
```

(%o20) $4\,\%e^{t}\left(\frac{d^3}{d\,t^3}\,y\right)-16\,\%e^{t}\left(\frac{d^2}{d\,t^2}\,y\right)+16\,\%e^{t}\left(\frac{d}{d\,t}\,y\right)=1$

```
(%i21) eq1:%/4/exp(t),expand;
```

(%o21) $\frac{d^3}{d\,t^3}\,y-4\left(\frac{d^2}{d\,t^2}\,y\right)+4\left(\frac{d}{d\,t}\,y\right)=\frac{\%e^{-t}}{4}$

```
(%i22) odeL(eq1,y,t);
```

(%o22) $y=t\,\%e^{2\,t}\,C3+\%e^{2\,t}\,C2+C1+\frac{\%e^{-t}\,(3\,t\,\%e^{3\,t}-2)}{72}$

```
(%i23) subst(t=log(x),%),expand;
```

(%o23) $y=x^2\log(x)\,C3+x^2\,C2+C1+\frac{x^2\log(x)}{24}-\frac{1}{36\,x}$

```
(%i24) y=collectterms(rhs(%),log(x),x);
```

(%o24) $y=x^2\log(x)\left(C3+\frac{1}{24}\right)+x^2\,C2+C1-\frac{1}{36\,x}$

```
(%i25) sol:subst([C1=%k1,C2=%k2,C3=%k3-1/24],%);
```

(%o25) $y=\%k3\,x^2\log(x)+\%k2\,x^2-\frac{1}{36\,x}+\%k1$

dchange

Function: dchange(tr,eq,y,x,new_func,new_var)
Makes the change tr:x=f(new_var) of independent variable x.

```
(%i1) load(odes)$ load(contrib_ode)$
```

1. (1-x^2)y'' - xy' + n^2y = 0

```
(%i3) eq:(1-x^2)*diff(y(x),x,2)-x*diff(y(x),x)+n^2*y(x)=0$
```

```
(%i4) assume(n>0)$
```

```
(%i5) tr:x=cos(t); itr:t=acos(x);
```

(%o5) $x=\cos(t)$

(%o6) $t=\operatorname{acos}(x)$

```
(%i7) dchange(tr,eq,y(x),x,y(t),t)$ eq1:trigsimp(%);
```

(%o8) $\frac{d^2}{d\,t^2}\,y(t)+n^2\,y(t)=0$

```
(%i9) ode2(%,y(t),t);
```

(%o9) $y(t)=\%k1\,\sin(n\,t)+\%k2\,\cos(n\,t)$

```
(%i10) sol:dchange(itr,%,y(t),t,y(x),x);
```

(%o10) $y(x)=\%k1\,\sin(n\,\operatorname{acos}(x))+\%k2\,\cos(n\,\operatorname{acos}(x))$

2. xy'' + y'/2-y = 0

```
(%i11) eq:x*diff(y(x),x,2)+diff(y(x),x)/2-y(x)=0$
```

```
(%i12) tr:x=t^2/4; itr:solve(%,t)[2];
```

(%o12) $x=\frac{t^2}{4}$

(%o13) $t=2\sqrt{x}$

```
(%i14) eq1:dchange(tr,eq,y(x),x,y(t),t),ratsimp;
```

(%o14) $\frac{d^2}{d\,t^2}\,y(t)-y(t)=0$

```
(%i15) ode2(%,y(t),t);
```

(%o15) $y(t)=\%k1\,\%e^{t}+\%k2\,\%e^{-t}$

```
(%i16) sol:dchange(itr,%,y(t),t,y(x),x);
```

(%o16) $y(x)=\%k1\,\%e^{2\sqrt{x}}+\%k2\,\%e^{-2\sqrt{x}}$

3. x^2y'' - 2xy' + 2y = x^5*log(x)

```
(%i17) eq:x^2*diff(y(x),x,2)-2*x*diff(y(x),x)+2*y(x)=x^5*log(x);
```

(%o17) $x^2\left(\frac{d^2}{d\,x^2}\,y(x)\right)-2\,x\left(\frac{d}{d\,x}\,y(x)\right)+2\,y(x)=x^5\,\log(x)$

```
(%i18) tr:x=exp(t); itr:solve(%,t)[1];
```

(%o18) $x=\%e^{t}$

(%o19) $t=\log(x)$

```
(%i20) eq1:dchange(tr,eq,y(x),x,y(t),t);
```

(%o20) $\frac{d^2}{d\,t^2}\,y(t)-3\left(\frac{d}{d\,t}\,y(t)\right)+2\,y(t)=t\,\%e^{5\,t}$

```
(%i21) ode2(%,y(t),t);
```

(%o21) $y(t)=\frac{(12\,t-7)\,\%e^{5\,t}}{144}+\%k1\,\%e^{2\,t}+\%k2\,\%e^{t}$

```
(%i22) sol:dchange(itr,%,y(t),t,y(x),x);
```

(%o22) $y(x)=\frac{x^5\,(12\,\log(x)-7)}{144}+\%k1\,x^2+\%k2\,x$

4. kamke 2.284

```
(%i23) eq:(2*x+1)^2*diff(y(x),x,2)-2*(2*x+1)*diff(y(x),x)-12*y(x)=3*x+1$
```

```
(%i24) tr:x=(%e^t-1)/2; itr:solve(tr,t)[1];
```

(%o24) $x=\frac{\%e^{t}-1}{2}$

(%o25) $t=\log(2\,x+1)$

```
(%i26) eq1:dchange(tr,eq,y(x),x,y(t),t);
```

(%o26) $4\left(\frac{d^2}{d\,t^2}\,y(t)\right)-8\left(\frac{d}{d\,t}\,y(t)\right)-12\,y(t)=\frac{3\,(\%e^{t}-1)}{2}+1$

```
(%i27) ode2(eq1,y(t),t);
```

(%o27) $y(t)=\%k1\,\%e^{3\,t}-\frac{9\,\%e^{t}-4}{96}+\%k2\,\%e^{-t}$

```
(%i28) sol:dchange(itr,%,y(t),t,y(x),x);
```

(%o28) $y(x)=\%k1\,(2\,x+1)^3+\frac{\%k2}{2\,x+1}-\frac{9\,(2\,x+1)-4}{96}$

```
(%i29) y(x)=map(factor,rhs(sol));
```

(%o29) $y(x)=\%k1\,(2\,x+1)^3+\frac{\%k2}{2\,x+1}-\frac{18\,x+5}{96}$

5.

(%i30) `eq:diff(y(x),x,2)-diff(y(x),x)+exp(4*x)*y(x)=0;`

(%o30) $\frac{d^2}{d x^2} y(x) - \frac{d}{d x} y(x) + \%e^{4x} y(x) = 0$

(%i31) `tr:x=log(t)/4;`

(%o31) $x = \frac{\log(t)}{4}$

(%i32) `itr:solve(tr,t)[1];`

(%o32) $t = \%e^{4x}$

(%i33) `eq1:dchange(tr,eq,y(x),x,y(t),t);`

(%o33) $16\,t^2 \left(\frac{d^2}{d t^2} y(t)\right) + 12\,t \left(\frac{d}{d t} y(t)\right) + t\, y(t) = 0$

(%i34) `eq2:subst(y(t)=y,eq1);`

(%o34) $16\,t^2 \left(\frac{d^2}{d t^2} y\right) + 12\,t \left(\frac{d}{d t} y\right) + t\, y = 0$

(%i35) `contrib_ode(eq2,y,t);`

(%o35) $[y = \text{bessel\_y}\left(\frac{1}{4}, \frac{\sqrt{t}}{2}\right) \%k2\, t^{1/8} + \text{bessel\_j}\left(\frac{1}{4}, \frac{\sqrt{t}}{2}\right) \%k1\, t^{1/8}]$

(%i36) `sol:subst(itr,%[1]);`

(%o36) $y = \text{bessel\_y}\left(\frac{1}{4}, \frac{\%e^{2x}}{2}\right) \%k2\, \%e^{x/2} + \text{bessel\_j}\left(\frac{1}{4}, \frac{\%e^{2x}}{2}\right) \%k1\, \%e^{x/2}$

odeC

Function: odeC(eq,r,x)
Solves ODE in respect to expression r.

Examples:

```
(%i1) load(odes)$
```

1. Bernoulli differential equation

```
(%i2) eq:'diff(y,x)+2*y/(x+1)=2*sqrt(y)/(x+1);
```

(%o2) $\frac{d}{d\,x}\,y+\frac{2\,y}{x+1}=\frac{2\,\sqrt{y}}{x+1}$

```
(%i3) odeC(eq,sqrt(y),x);
```

(%o3) $\sqrt{y}=\frac{x}{x+1}+\frac{\%c}{x+1}$

```
(%i4) ode2(eq,y,x);
```

(%o4) $-\log(\sqrt{y}-1)=\log(x+1)+\%c$

2. boj 360.

```
(%i5) eq:x*'diff(y,x,3)-'diff(y,x,2)-x*'diff(y,x)+y=-2*x^3;
```

(%o5) $x\left(\frac{d^3}{d\,x^3}\,y\right)-\frac{d^2}{d\,x^2}\,y-x\left(\frac{d}{d\,x}\,y\right)+y=-2\,x^3$

```
(%i6) odeC(eq,'diff(y,x,2)+y,x);
```

(%o6) $\frac{d^2}{d\,x^2}\,y+y=2\,y-x^3+\%c\,x$

```
(%i7) ode2(%,y,x);
```

(%o7) $y=\%k1\,\%e^{x}+\%k2\,\%e^{-x}+x^3+(6-\%c)\,x$

```
(%i8) sol:subst(%c=6-%k3,%);
```

(%o8) $y=\%k1\,\%e^{x}+\%k2\,\%e^{-x}+x^3+\%k3\,x$

3. sam 4.35.

```
(%i9) eq1:'diff(x,t)=y+z$
      eq2:'diff(y,t)=x+z$
      eq3:'diff(z,t)=x+y$
```

```
(%i12) odeC(eq1+eq2+eq3,x+y+z,t)$
       s1:subst(%c=3*C1,%);
```

(%o13) $z+y+x=3\,\%e^{2\,t}\,C1$

```
(%i14) odeC(eq1-eq2,y-x,t)$
       s2:subst(%c=3*C2,%);
```
(%o15) $y-x=3\,\%e^{-t}\,C2$

```
(%i16) odeC(eq2-eq3,z-y,t)$
       s3:subst(%c=3*C3,%);
```
(%o17) $z-y=3\,\%e^{-t}\,C3$

```
(%i18) sol:solve([s1,s2,s3],[x,y,z])[1],expand;
```
(%o18) $[x=-\%e^{-t}\,C3-2\,\%e^{-t}\,C2+\%e^{2\,t}\,C1, y=-\%e^{-t}\,C3+\%e^{-t}\,C2+\%e^{2\,t}\,C1, z=2\,\%e^{-t}\,C3+\%e^{-t}\,C2+\%e^{2\,t}\,C1]$

Test:

```
(%i19) subst(sol,[eq1,eq2,eq3])$
       ev(%, nouns)$
       makelist(rhs(%[k])-lhs(%[k]),k,1,3);
```
(%o21) [0,0,0]

4. filipov 65.

```
(%i22) eq:'diff(y,x)=sqrt(4*x+2*y-1);
```
(%o22) $\frac{d}{d\,x}\,y=\sqrt{2\,y+4\,x-1}$

```
(%i23) ode2(eq,y,x);
```
(%o23) false

```
(%i24) load(contrib_ode)$
```

```
(%i25) contrib_ode(eq,y,x);
```
Is p positive, negative or zero? p;

(%o25) $[-\frac{-8\,\log(\sqrt{2\,y+4\,x-1}+2)+4\,\sqrt{2\,y+4\,x-1}-4\,x+1}{4}=\%c]$

```
(%i26) odeC(eq,2*y+4*x-1,x);
```
(%o26) $-2\,\log(\sqrt{2\,y+4\,x-1}+2)+\sqrt{2\,y+4\,x-1}+2=x+\%c$

solvet

Function: solvet(eq,x)
Returns rectform solution of polynomial equation.
In "casus irreducibilis" give real solutions expressed
in trigonometric functions.

One version of "solvet" is:

```
(%i1) solvet(eq,x):=block([polf,spr,k],
      spr:solve(eq,x),
      polf(x):=block([rx],
      rx:rectform(x),
      if freeof(%i,x) or atom(x) or
      freeof(sin,rx) then return(rx) else
      map(polarform,x),
      rectform(%%),
      trigsimp(%%),
      trigreduce(%%)),
      makelist(x=polf(rhs(spr[k])),k,1,length(spr)),
      sort(%%)
      )$
```

Examples:

```
(%i2) solvet(x^3-3*x^2+1,x);
```

(%o2) $[x=2\cos\left(\frac{\pi}{9}\right)+1, x=2\cos\left(\frac{5\pi}{9}\right)+1, x=2\cos\left(\frac{7\pi}{9}\right)+1]$

```
(%i3) solvet(x^6-3*x^5-3*x^4+12*x^3-3*x^2-6*x+2=0,x);
```

(%o3) $[x=1, x=1-\sqrt{3}, x=\sqrt{3}+1, x=2\cos\left(\frac{2\pi}{9}\right), x=2\cos\left(\frac{4\pi}{9}\right), x=2\cos\left(\frac{8\pi}{9}\right)]$

```
(%i4) solvet(x^3-15*x-5,x);
```

(%o4) $[x=2\sqrt{5}\cos\left(\frac{\operatorname{atan}(\sqrt{19})}{3}-\frac{2\pi}{3}\right), x=2\sqrt{5}\cos\left(\frac{\operatorname{atan}(\sqrt{19})}{3}+\frac{2\pi}{3}\right), x=2\sqrt{5}\cos\left(\frac{\operatorname{atan}(\sqrt{19})}{3}\right)]$

```
(%i5) solve(x^3-15*x-5,x);
```

(%o5) $[x=\left(-\frac{\sqrt{3}\,\%i}{2}-\frac{1}{2}\right)\left(\frac{5\sqrt{19}\,\%i}{2}+\frac{5}{2}\right)^{1/3}+\frac{5\left(\frac{\sqrt{3}\,\%i}{2}-\frac{1}{2}\right)}{\left(\frac{5\sqrt{19}\,\%i}{2}+\frac{5}{2}\right)^{1/3}}, x=\left(\frac{\sqrt{3}\,\%i}{2}-\frac{1}{2}\right)\left(\frac{5\sqrt{19}\,\%i}{2}+\frac{5}{2}\right)^{1/3}+\frac{5\left(-\frac{\sqrt{3}\,\%i}{2}-\frac{1}{2}\right)}{\left(\frac{5\sqrt{19}\,\%i}{2}+\frac{5}{2}\right)^{1/3}}, x=\left(\frac{5\sqrt{19}\,\%i}{2}+\frac{5}{2}\right)^{1/3}+\frac{5}{\left(\frac{5\sqrt{19}\,\%i}{2}+\frac{5}{2}\right)^{1/3}}]$

ode1_ic

```
Function:  ode1_ic(eqn, dvar, ivar, ic)
The function ode1_ic solves an ordinary differential equation(ODE)
of first order with initial condition  y(x0) = y0.
Here ic is list [x0,y0].
```

Examples:

```
(%i1) load(odes)$
```

1. x^2y'+3xy = sin(x)/x, y(pi) = 0.

```
(%i2) ode1_ic(x^2*'diff(y,x) + 3*y*x = sin(x)/x,y,x,[%pi,0]);
```

(%o2) $y=-\frac{\cos(x)+1}{x^3}$

2.   (y^4e^y+2x)y' = y, y(0) = 1.

```
(%i3) eq:(y^4*exp(y)+2*x)*'diff(y,x)=y$
```

```
(%i4) ode1_ic(eq,y,x,[0,1]);
```

(%o4) $\frac{(y^3-y^2)\%e^y-x}{y^2}=0$

```
(%i5) solve(%,x);
```

(%o5) $[x=(y^3-y^2)\%e^y]$

3. xy' + y = 2y^2log(x), y(1)=1/2.

```
(%i6) eq:x*'diff(y,x)+y=2*y^2*log(x)$
```

```
(%i7) ode1_ic(eq,y,x,[1,1/2]);
```

(%o7) $y=\frac{1}{2\log(x)+2}$

4. (x^2-1)y' + 2xy^2 = 0, y(0) = 1.

```
(%i8) eq:(x^2-1)*'diff(y,x)+2*x*y^2=0$
```

```
(%i9) ode1_ic(eq,y,x,[0,1]);
```

(%o9) $y=\frac{1}{\log(1-x^2)+1}$

ode2_ic

Function: ode2_ic(eqn, dvar, ivar, ic)
The function ode2_ic solve an ordinary differential equation(ODE) of second order with initial conditions y(x0) = y0, y'(x0) = y1. Here ic is list [x0, y0, y1].

Examples:

```
(%i1) load(odes)$
```

1. y'' + yy'^3=0, y(0)=0, y'(0)=2

```
(%i2) eq:'diff(y,x,2) + y*'diff(y,x)^3 = 0$
(%i3) sol:ode2_ic(eq,y,x,[0,0,2]);
```

(%o3) $y=(\sqrt{9x^2+1}+3x)^{1/3}-\frac{1}{(\sqrt{9x^2+1}+3x)^{1/3}}$

Test:

```
(%i4) ev(rhs(sol),x=0);
(%o4) 0
(%i5) diff(rhs(sol),x)$ ev(%,x=0);
(%o6) 2
(%i7) subst(sol,eq)$
      ev(%, nouns)$
      radcan(%);
(%o9) 0=0
```

2. y'' = 128*y^3, y(0) = 1, y'(0) = 8.

```
(%i10) eq:'diff(y,x,2)=128*y^3$
(%i11) ode2_ic(eq,y,x,[0,1,8]);
```

(%o11) $y=-\frac{1}{8x-1}$

3. y'' + y = 1/cos(x), y(0)=1, y'(0)=0.

```
(%i12) eq:'diff(y,x,2)+y = 1/cos(x)$
(%i13) sol:ode2_ic(eq,y,x,[0,1,0]);
```

(%o13) $y=\cos(x)\log(\cos(x))+x\sin(x)+\cos(x)$

P_iter

Function: P_iter(eq, x, y, x0, y0, n).
Solves first order differential equation using Picard iterative process.
http://www.sosmath.com/diffeq/first/picard/picard.html

```
(%i10) load(odes)$
```

Examples:

1. y'=x^2+y^2, y(0)=0

```
(%i11) eq:'diff(y,x)=x^2+y^2$
```

```
(%i12) x0:0$ y0:0$
```

```
(%i14) for k:0 thru 3 do
       print(P_iter(eq,x,y,x0,y0,k))$
```

$$0$$
$$\frac{x^3}{3}$$
$$\frac{x^7}{63}+\frac{x^3}{3}$$
$$\frac{x^{15}}{59535}+\frac{2\,x^{11}}{2079}+\frac{x^7}{63}+\frac{x^3}{3}$$

2. y' = 2x(1 + y), y(0) = 0.

```
(%i15) eq:'diff(y,x)=2*x*(1+y)$
```

```
(%i16) x0:0$ y0:0$
```

```
(%i18) for k:0 thru 5 do
       print(P_iter(eq,x,y,x0,y0,k))$
```

$$0$$
$$x^2$$
$$\frac{x^4}{2}+x^2$$
$$\frac{x^6}{6}+\frac{x^4}{2}+x^2$$
$$\frac{x^8}{24}+\frac{x^6}{6}+\frac{x^4}{2}+x^2$$
$$\frac{x^{10}}{120}+\frac{x^8}{24}+\frac{x^6}{6}+\frac{x^4}{2}+x^2$$

ode1taylor

Function: ode1taylor(eq, x0, y0, n).
Solves first order differential equation using Taylor-series expansion.

```
(%i1) load(odes)$
```

1. y'=x+y^2, y(0)=1

```
(%i2) eq:diff(y(x),x)=x+y(x)^2;
```

(%o2) $\frac{d}{d\,x}\,y(x)=y(x)^2+x$

```
(%i3) ode1taylor(eq,0,1,6);
```

(%o3)/T/ $1+x+\frac{3\,x^2}{2}+\frac{4\,x^3}{3}+\frac{17\,x^4}{12}+\frac{31\,x^5}{20}+\frac{149\,x^6}{90}+...$

2. y'=x^2+y^2, y(0)=0

```
(%i4) eq:diff(y(x),x)=x^2+y(x)^2;
```

(%o4) $\frac{d}{d\,x}\,y(x)=y(x)^2+x^2$

```
(%i5) ode1taylor(eq,0,0,15);
```

(%o5)/T/ $\frac{x^3}{3}+\frac{x^7}{63}+\frac{2\,x^{11}}{2079}+\frac{13\,x^{15}}{218295}+...$

3. y'=x-y^2, y(1)=-1

```
(%i6) eq:diff(y(x),x)=x-y(x)^2;
```

(%o6) $\frac{d}{d\,x}\,y(x)=x-y(x)^2$

```
(%i7) ode1taylor(eq,1,-1,5);
```

(%o7)/T/ $-1+\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}+\frac{(x-1)^4}{6}+\frac{(x-1)^5}{60}+...$

ode2taylor

Function: ode2taylor(eq, x0, y0, y1, n).
Solves second order differential equation using Taylor-series expansion.

```
(%i1) load(odes)$
```

Examples:

1. Airy's Equation y''-xy=0, y(0)=1, y'(0)=0.

```
(%i2) eq:'diff(y(x),x,2)-x*y(x)=0;
```

(%o2) $\frac{d^2}{d x^2} y(x) - x\, y(x) = 0$

```
(%i3) ode2taylor(eq,0,1,0,15);
```

(%o3)/T/ $1 + \frac{x^3}{6} + \frac{x^6}{180} + \frac{x^9}{12960} + \frac{x^{12}}{1710720} + \frac{x^{15}}{359251200} + ...$

2. y''=(y')^2+xy, y(1)=1, y'(0)=0

```
(%i4) eq:'diff(y(x),x,2)='diff(y(x),x,1)^2+x*y(x);
```

(%o4) $\frac{d^2}{d x^2} y(x) = \left(\frac{d}{d x} y(x)\right)^2 + x\, y(x)$

```
(%i5) ode2taylor(eq,1,1,0,5);
```

(%o5)/T/ $1 + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{6} + \frac{(x-1)^4}{8} + \frac{(x-1)^5}{12} + ...$

3.

```
(%i6) eq:'diff(y(x),x,2)+x*'diff(y(x),x)+y(x)=0;
```

(%o6) $\frac{d^2}{d x^2} y(x) + x \left(\frac{d}{d x} y(x)\right) + y(x) = 0$

```
(%i7) ode2taylor(eq,0,0,1,15);
```

(%o7)/T/ $x - \frac{x^3}{3} + \frac{x^5}{15} - \frac{x^7}{105} + \frac{x^9}{945} - \frac{x^{11}}{10395} + \frac{x^{13}}{135135} - \frac{x^{15}}{2027025} + ...$

```
(%i8) sum((-1)^n*2^n*n!*x^(2*n+1)/(2*n+1)!,n,0,7);
```

(%o8) $-\frac{x^{15}}{2027025} + \frac{x^{13}}{135135} - \frac{x^{11}}{10395} + \frac{x^9}{945} - \frac{x^7}{105} + \frac{x^5}{15} - \frac{x^3}{3} + x$

ode1exact

(%i1) load(odes)$

Function: ode1exact(eq).
Solves first order exact equation.
http://www.math24.net/exact-equations.html

Examples:

1.

(%i2) eq:2*x*y*dx+(x^2+3*y^2)*dy=0;

(%o2) $dy\,(3\,y^2+x^2)+2\,dx\,x\,y=0$

(%i3) ode1exact(eq);

(%o3) $y^3+x^2\,y=C$

2.

(%i4) eq:(6*x^2-y+3)*dx+(3*y^2-x-3)*dy=0;

(%o4) $dy\,(3\,y^2-x-3)+dx\,(-y+6\,x^2+3)=0$

(%i5) ode1exact(eq);

(%o5) $y^3-x\,y-3\,y+2\,x^3+3\,x=C$

3.

(%i6) eq:exp(y)*dx+(2*y+x*exp(y))*dy=0;

(%o6) $dy\,(x\,\%e^y+2\,y)+dx\,\%e^y=0$

(%i7) ode1exact(eq);

(%o7) $x\,\%e^y+y^2=C$

4.

(%i8) eq:(x*dx+y*dy)/sqrt(x^2+y^2)+(x*dy-y*dx)/x^2=0;

(%o8) $\frac{dy\,y+dx\,x}{\sqrt{y^2+x^2}}+\frac{dy\,x-dx\,y}{x^2}=0$

(%i9) ode1exact(eq);

(%o9) $\sqrt{y^2+x^2}+\frac{y}{x}=C$

intfactor1

Function: intfactor(eq, omega).
Find intfactor mu = mu(omega) of the first order differential equation.
http://www.math24.net/using-integrating-factor.html

Examples:

```
(%i1) load(odes)$
```

1.

```
(%i2) eq:(1+y^2)*dx+x*y*dy=0$
```

```
(%i3) intfactor1(eq,x);
```

(%o3) x

```
(%i4) ode1exact(eq*%);
```

(%o4) $\frac{x^2 y^2}{2}+\frac{x^2}{2}=C$

2.

```
(%i5) eq:(x*y^2-2*y^3)*dx+(3-2*x*y^2)*dy=0$
```

```
(%i6) intfactor1(eq,y);
```

(%o6) $\frac{1}{y^2}$

```
(%i7) ode1exact(eq*%);
```

(%o7) $-2\,x\,y-\frac{3}{y}+\frac{x^2}{2}=C$

3.

```
(%i8) eq:y*dx+(x^2+y^2-x)*dy=0$
```

```
(%i9) ode1exact(eq);
```

(%o9) false

```
(%i10) intfactor1(eq,x^2+y^2);
```

(%o10) $\frac{1}{y^2+x^2}$

```
(%i11) ode1exact(eq*%);
```

(%o11) $y+\operatorname{atan}\left(\frac{x}{y}\right)=C$

4.

```
(%i12) eq:x*y*dx+(2*x^2+3*y^2-20)*dy=0$
(%i13) intfactor1(eq,y);
```

(%o13) y^3

```
(%i14) ode1exact(eq*%);
```

(%o14) $\frac{y^6}{2}+\frac{x^2\,y^4}{2}-5\,y^4=C$

5.

```
(%i15) eq:(x^2*y^3+6*y^5)*dx+(2*x^3*y^2+12*x^4)*dy=0$
(%i16) intfactor1(eq/y,x*y);
```

(%o16) $\frac{1}{x^4\,y^4}$

```
(%i17) ode1exact(-eq/y*%);
```

(%o17) $\frac{1}{x\,y^2}+\frac{3}{y^4}+\frac{2}{x^3}=C$

Other method:

```
(%i18) mu:x^a*y^b;
```

(%o18) $x^a\,y^b$

```
(%i19) diff(mu*(x^2*y^3+6*y^5),y)=diff(mu*(2*x^3*y^2+12*x^4),x)$
(%i20) factor(lhs(%)-rhs(%));
```

(%o20) $x^a\,y^b(6\,b\,y^4+30\,y^4+b\,x^2\,y^2-2\,a\,x^2\,y^2-3\,x^2\,y^2-12\,a\,x^3-48\,x^3)$

```
(%i21) collectterms(%/mu,x,y);
```

(%o21) $(6\,b+30)y^4+(b-2\,a-3)x^2\,y^2+(-12\,a-48)x^3$

```
(%i22) solve([coeff(%,y^4),coeff(%,x^2*y^2),coeff(%,x^3)],[a,b]);
solve: dependent equations eliminated: (2)
```

(%o22) $[[a=-4,b=-5]]$

```
(%i23) 'mu=subst(%[1],mu);
```

(%o23) $\mu=\frac{1}{x^4\,y^5}$

odeL

Function: odeL(eqn, dvar, ivar)
The function odeL solves an linear ODEs with constant coefficients.

Examples:

```
(%i1) load(odes)$ load(contrib_ode)$
```

1. y''' - 2y'' + y' = 0.

```
(%i3) eq:'diff(y,x,3)-2*'diff(y,x,2)+'diff(y,x) = 0$
(%i4) odeL(eq,y,x);
```

(%o4) $y = x\,\%e^{x}\,C3 + \%e^{x}\,C2 + C1$

2. y'''' + 8y'' + 16y = x exp(3x) + sin(x)^2 + 1

```
(%i5) eq:'diff(y,x,4)+8*'diff(y,x,2)+16*y=x*exp(3*x)+sin(x)^2+1$
(%i6) sol:odeL(eq,y,x);
```

(%o6) $y = x\sin(2x)C4 + \sin(2x)C3 + x\cos(2x)C2 + \cos(2x)C1 + \frac{2197x^2\cos(2x) + (832x - 768)\%e^{3x} + 13182}{140608}$

```
(%i7) ode_check(eq,sol);
```

(%o7) 0

3. y'''- 3y'' + y = sin(x)^3.

```
(%i8) eq:'diff(y,x,3)-3*'diff(y,x,2)+y=sin(x)^3;
```

(%o8) $\frac{d^3}{dx^3}y - 3\left(\frac{d^2}{dx^2}y\right) + y = \sin(x)^3$

```
(%i9) solvet(k^3-3*k^2+1=0,k);
```

(%o9) $[k = 2\cos\left(\frac{5\pi}{9}\right) + 1, k = 2\cos\left(\frac{7\pi}{9}\right) + 1, k = 2\cos\left(\frac{\pi}{9}\right) + 1]$

```
(%i10) sol:odeL(eq,y,x);
```

(%o10) $y = \%e^{2\cos\left(\frac{7\pi}{9}\right)x + x}C3 + \%e^{2\cos\left(\frac{5\pi}{9}\right)x + x}C2 + \%e^{2\cos\left(\frac{\pi}{9}\right)x + x}C1 - \frac{28\sin(3x) + 27\cos(3x) - 1068\sin(x) - 267\cos(x)}{6052}$

```
(%i11) ode_check(eq,sol);
```

(%o11) 0

odeL_ic

Function: odeL_ic(eqn, dvar, ivar, ic)
The function odeL_ic solves initial value problems for linear ODEs with constant coefficients.

Examples:

```
(%i1) load(odes)$ load(contrib_ode)$
```

1. y''' + y'' = x + exp(-x), y(0) = 1, y'(0) = 0, y''(0) = 1.

```
(%i3) eq:'diff(y,x,3)+'diff(y,x,2)=x + exp(-x);
```

(%o3) $\frac{d^3}{d x^3} y+\frac{d^2}{d x^2} y=\%e^{-x}+x$

```
(%i4) odeL_ic(eq, y, x, [0, 1, 0, 1]),expand;
```

(%o4) $y=x\ \%e^{-x}+4\ \%e^{-x}+\frac{x^3}{6}-\frac{x^2}{2}+3\ x-3$

2. y''''-y=8*exp(x), y(0)=0, y'(0)=2, y''(0)=4, y'''(0)=6.

```
(%i5) eq:'diff(y,x,4)-y=8*exp(x);
```

(%o5) $\frac{d^4}{d x^4} y-y=8\ \%e^{x}$

```
(%i6) odeL_ic(eq, y, x, [0, 0, 2, 4, 6]);
```

(%o6) $y=2\ x\ \%e^{x}$

3. y''''' - y' = 0, y(0)=0, y'(0)=1, y''(0)=0, y'''(0)=1, y''''(0)=2.

```
(%i7) eq:'diff(y,x,5)-'diff(y,x)=0;
```

(%o7) $\frac{d^5}{d x^5} y-\frac{d}{d x} y=0$

```
(%i8) sol:odeL_ic(eq,y,x,[0,0,1,0,1,2]);
```

(%o8) $y=\cos(x)+\%e^{x}-2$

Test:

```
(%i9) ode_check(eq,sol);
(%o9) 0
```

```
(%i10) makelist(diff(rhs(sol),x,k),k,0,4)$
       ev(%,x=0);
(%o11) [0,1,0,1,2]
```

fs

Function: fs(eq, y, x).
Find fundamental system of solutions of the linear n-th order differential equation with constant coeficients.

Examples:

(%i1) `load(odes)$`

1.

(%i2) `eq:'diff(y,x,3)+3*'diff(y,x,2)-10*'diff(y,x)=x-1;`

(%o2) $$\frac{d^3}{d\,x^3}\,y+3\left(\frac{d^2}{d\,x^2}\,y\right)-10\left(\frac{d}{d\,x}\,y\right)=x-1$$

(%i3) `fs(eq,y,x);`

(%o3) $[1, \%e^{-5\,x}, \%e^{2\,x}]$

(%i4) `odeL(eq,y,x);`

(%o4) $$y=\%e^{2\,x}\,C3+\%e^{-5\,x}\,C2+C1-\frac{5\,x^2-7\,x}{100}$$

2.

(%i5) `eq:'diff(y,x,4)+8*'diff(y,x,2)+16*y=x^2*exp(x)*sin(x);`

(%o5) $$\frac{d^4}{d\,x^4}\,y+8\left(\frac{d^2}{d\,x^2}\,y\right)+16\,y=x^2\,\%e^x\,\sin(x)$$

(%i6) `fs(eq,y,x);`

(%o6) $[\cos(2\,x), x\cos(2\,x), \sin(2\,x), x\sin(2\,x)]$

(%i7) `sol:odeL(eq,y,x);`

(%o7) $$y=x\sin(2\,x)\,C4+\sin(2\,x)\,C3+x\cos(2\,x)\,C2+\cos(2\,x)\,C1+\frac{(75\,x^2-260\,x+268)\,\%e^x\sin(x)+(-100\,x^2+180\,x+26)\,\%e^x\cos(x)}{2500}$$

3.

(%i8) `eq:'diff(y,x,8)+'diff(y,x,2)=x^5$`

(%i9) `solvet(k^8+k^2=0,k);`

(%o9) $$\left[k=\%i, k=\frac{\%i}{2}-\frac{\sqrt{3}}{2}, k=-\frac{\%i}{2}-\frac{\sqrt{3}}{2}, k=-\%i, k=\frac{\sqrt{3}}{2}-\frac{\%i}{2}, k=\frac{\%i}{2}+\frac{\sqrt{3}}{2}, k=0\right]$$

```
(%i10) fs(eq,y,x);
```

(%o10) $\left[1, x, \%e^{-\frac{\sqrt{3}\,x}{2}} \cos\left(\frac{x}{2}\right), \%e^{\frac{\sqrt{3}\,x}{2}} \cos\left(\frac{x}{2}\right), \%e^{-\frac{\sqrt{3}\,x}{2}} \sin\left(\frac{x}{2}\right), \%e^{\frac{\sqrt{3}\,x}{2}} \sin\left(\frac{x}{2}\right), \cos(x), \sin(x)\right]$

```
(%i11) sol:odeL(eq,y,x);
```

(%o11) $y=\sin(x)\,C8+\cos(x)\,C7+\%e^{\frac{\sqrt{3}\,x}{2}} \sin\left(\frac{x}{2}\right) C6+\%e^{-\frac{\sqrt{3}\,x}{2}} \sin\left(\frac{x}{2}\right) C5+\%e^{\frac{\sqrt{3}\,x}{2}} \cos\left(\frac{x}{2}\right) C4+\%e^{-\frac{\sqrt{3}\,x}{2}} \cos\left(\frac{x}{2}\right) C3+x\,C2+C1+\frac{x^7-5040\,x}{42}$

Test:

```
(%i12) load(contrib_ode)$
```

```
(%i13) ode_check(eq,sol);
(%o13) 0
```

4.

```
(%i14) eq:'diff(y,x,6)-3*'diff(y,x,5)-3*'diff(y,x,4)+12*'diff(y,x,3)
-3*'diff(y,x,2)-6*'diff(y,x,1)+2*y=2*x^7+sin(x)^3;
```

(%o14) $\frac{d^6}{d\,x^6}\,y-3\left(\frac{d^5}{d\,x^5}\,y\right)-3\left(\frac{d^4}{d\,x^4}\,y\right)+12\left(\frac{d^3}{d\,x^3}\,y\right)-3\left(\frac{d^2}{d\,x^2}\,y\right)-6\left(\frac{d}{d\,x}\,y\right)+2\,y=\sin(x)^3+2\,x^7$

```
(%i15) fs(eq,y,x);
```

(%o15) $\left[\%e^{x}, \%e^{2\cos\left(\frac{2\,\pi}{9}\right) x}, \%e^{2\cos\left(\frac{4\,\pi}{9}\right) x}, \%e^{2\cos\left(\frac{8\,\pi}{9}\right) x}, \%e^{x-\sqrt{3}\,x}, \%e^{\sqrt{3}\,x+x}\right]$

```
(%i16) sol:odeL(eq,y,x)$
```

```
(%i17) expand(%);
```

(%o17) $y=\%e^{\sqrt{3}\,x+x}\,C6+\%e^{x-\sqrt{3}\,x}\,C5+\%e^{2\cos\left(\frac{8\,\pi}{9}\right) x}\,C4+\%e^{2\cos\left(\frac{4\,\pi}{9}\right) x}\,C3+\%e^{2\cos\left(\frac{2\,\pi}{9}\right) x}\,C2+\%e^{x}\,C1+\frac{943\sin(3\,x)}{8145160}-\frac{1071\cos(3\,x)}{8145160}+\frac{3\sin(x)}{1768}+\frac{63\cos(x)}{1768}+x^7+21\,x^6+441\,x^5+6300\,x^4+74970\,x^3+644490\,x^2+3734010\,x+10735200$

Test:

```
(%i18) ode_check(eq,sol)$
trigreduce(%)$
trigrat(%);
(%o20) 0
```

partsol

Function: partsol(eq, y, x).
Find partial solution of the linear n-th order differential equation with constant coefficients.

Examples:

```
(%i1) load(odes)$
```

1.

```
(%i2) eq:'diff(y,x,3)+3*'diff(y,x,2)-10*'diff(y,x)=x-1;
```

(%o2) $$\frac{d^3}{d x^3} y+3\left(\frac{d^2}{d x^2} y\right)-10\left(\frac{d}{d x} y\right)=x-1$$

```
(%i3) partsol(eq,y,x);
```

(%o3) $$-\frac{5 x^2-7 x}{100}$$

```
(%i4) odeL(eq,y,x);
```

(%o4) $$y=\%e^{2 x} C3+\%e^{-5 x} C2+C1-\frac{5 x^2-7 x}{100}$$

2.

```
(%i5) eq:'diff(y,x,4)+'diff(y,x,3)-3*'diff(y,x,2)-5*'diff(y,x)-2*y=
exp(2*x)-exp(-x);
```

(%o5) $$\frac{d^4}{d x^4} y+\frac{d^3}{d x^3} y-3\left(\frac{d^2}{d x^2} y\right)-5\left(\frac{d}{d x} y\right)-2 y=\%e^{2 x}-\%e^{-x}$$

```
(%i6) partsol(eq,y,x);
```

(%o6) $$\frac{\%e^{-x}(2 x \%e^{3 x}+3 x^3+4 x^2+2 x)}{54}$$

```
(%i7) odeL(eq,y,x);
```

(%o7) $$y=\%e^{2 x} C4+x^2 \%e^{-x} C3+x \%e^{-x} C2+\%e^{-x} C1+\frac{\%e^{-x}(2 x \%e^{3 x}+3 x^3+4 x^2+2 x)}{54}$$

```
(%i8) expand(%)$
```

```
(%i9) y=collectterms(rhs(%),exp(-x),exp(2*x));
```

(%o9) $$y=\%e^{2 x}\left(C4+\frac{x}{27}\right)+\%e^{-x}\left(x^2 C3+x C2+C1+\frac{x^3}{18}+\frac{2 x^2}{27}+\frac{x}{27}\right)$$

3.

```
(%i10) eq:'diff(y,x,3)+'diff(y,x,1)=1/cos(x)$
```

```
(%i11) partsol(eq,y,x);
```

(%o11) $$-\frac{\log\left(\frac{\sin(x)-1}{\sin(x)+1}\right)-2\sin(x)\log(\cos(x))+2x\cos(x)}{2}$$

```
(%i12) sol:odeL(eq,y,x);
```

(%o12) $$y=\sin(x)C3+\cos(x)C2+C1-\frac{\log\left(\frac{\sin(x)-1}{\sin(x)+1}\right)-2\sin(x)\log(\cos(x))+2x\cos(x)}{2}$$

Test:

```
(%i13) load(contrib_ode)$
```

```
(%i14) ode_check(eq,sol);
```

(%o14) 0

4.

```
(%i15) eq:'diff(y,x,3)+8*'diff(y,x,1)+9*y=cos(x)^3;
```

(%o15) $$\frac{d^3}{d\,x^3}y+8\left(\frac{d}{d\,x}y\right)+9\,y=\cos(x)^3$$

```
(%i16) partsol(eq,y,x);
```

(%o16) $$-\frac{13\sin(3x)-39\cos(3x)-63\sin(x)-81\cos(x)}{1560}$$

```
(%i17) sol:odeL(eq,y,x);
```

(%o17) $$y=\%e^{x/2}\sin\left(\frac{\sqrt{5}\sqrt{7}\,x}{2}\right)C3+\%e^{x/2}\cos\left(\frac{\sqrt{5}\sqrt{7}\,x}{2}\right)C2+\%e^{-x}C1-\frac{13\sin(3x)-39\cos(3x)-63\sin(x)-81\cos(x)}{1560}$$

Test:

```
(%i18) load(contrib_ode)$
```

```
(%i19) ode_check(eq,sol);
```

(%o19) 0

odeM

Function: odeM(A,F,t)
Find solutions of linear system of ODEs
with constant coefficients in matrix form:
Y' = AY + F

Examples:

```
(%i1) load(odes)$
```

1. Y' = AY + F.

```
(%i2) A:matrix([1,1],[4,1]);
```

(%o2) $\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$

```
(%i3) F:transpose([t-2,4*t-1]);
```

(%o3) $\begin{bmatrix} t-2 \\ 4t-1 \end{bmatrix}$

```
(%i4) sol:odeM(A,F,t);
```

(%o4) $\begin{bmatrix} \left(\frac{\%e^{3t}}{4}-\frac{\%e^{-t}}{4}\right)C2+\left(\frac{\%e^{3t}}{2}+\frac{\%e^{-t}}{2}\right)C1-t \\ \left(\frac{\%e^{3t}}{2}+\frac{\%e^{-t}}{2}\right)C2+(\%e^{3t}-\%e^{-t})C1+1 \end{bmatrix}$

Test:

```
(%i5) diff(sol,t)-A.sol-F$
      expand(%);
```

(%o6) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

2. Y' = AY.

```
(%i7) A:matrix([2,0,-8,-3],[-18,-1,0,0],[-9,-3,-25,-9],[33,10,90,32]);
```

(%o7) $\begin{bmatrix} 2 & 0 & -8 & -3 \\ -18 & -1 & 0 & 0 \\ -9 & -3 & -25 & -9 \\ 33 & 10 & 90 & 32 \end{bmatrix}$

```
(%i8) F:transpose([0,0,0,0])$
```

```
(%i9) charpoly(A, x),factor;
```

(%o9) $(x^2-4x+13)^2$

```
(%i10) solve(%);
```

(%o10) $[x=2-3\%i, x=3\%i+2]$

```
(%i11) odeM(A,F,t)$
```

```
(%i12) sol:ratsimp(%);
```

(%o12)

(-3 t-1)%e^(2 t) sin(3 t) C4+((-9 t-3)%e^(2 t) sin(3 t)+t %e^(2 t) cos(3 t)) C3

((9 t+3)%e^(2 t) sin(3 t)-9 t %e^(2 t) cos(3 t)) C4+((24 t+10)%e^(2 t) sin(3 t)-30 t %e^(2 t) cos(3 t)) C3+(3 t %

-3 %e^(2 t) sin(3 t) C4+(%e^(2 t) cos(3 t)-9 %e^(2 t) sin(3 t)) C

(9 %e^(2 t) sin(3 t)+(3 t+1)%e^(2 t) cos(3 t)) C4+((t+27)%e^(2 t) sin(3 t)+9 t %e^(2 t) cos(3 t)) C3+(

```
(%i13) sol[1,1];
```

(%o13) $(-3t-1)\%e^{2t}\sin(3t)C4+((-9t-3)\%e^{2t}\sin(3t)+t\%e^{2t}\cos(3t))C3-t\%e^{2t}\sin(3t)C2+(\%e^{2t}\cos(3t)-3t\%e^{2t}\sin(3t))C1$

```
(%i14) sol[2,1];
```

(%o14) $((9t+3)\%e^{2t}\sin(3t)-9t\%e^{2t}\cos(3t))C4+((24t+10)\%e^{2t}\sin(3t)-30t\%e^{2t}\cos(3t))C3+(3t\%e^{2t}\sin(3t)+(1-3t)\%e^{2t}\cos(3t))C2+((9t-3)\%e^{2t}\sin(3t)-9t\%e^{2t}\cos(3t))C1$

```
(%i15) sol[3,1];
```

(%o15) $-3\%e^{2t}\sin(3t)C4+(\%e^{2t}\cos(3t)-9\%e^{2t}\sin(3t))C3-\%e^{2t}\sin(3t)C2-3\%e^{2t}\sin(3t)C1$

```
(%i16) sol[4,1];
```

(%o16) $(9\%e^{2t}\sin(3t)+(3t+1)\%e^{2t}\cos(3t))C4+((t+27)\%e^{2t}\sin(3t)+9t\%e^{2t}\cos(3t))C3+(3\%e^{2t}\sin(3t)+t\%e^{2t}\cos(3t))C2+(10\%e^{2t}\sin(3t)+3t\%e^{2t}\cos(3t))C1$

Test:

```
(%i17) diff(sol,t)-A.sol$
expand(%);
```

(%o18) $\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

odeM_ic

Function: odeM_ic(A, F, t, t0, Y0)
Find solutions of initial problem for linear system of ODEs
in matrix form:

Y' = AY + F, Y(t0) = Y0.

(updated version of odelinsys)

Examples:

```
(%i1) load(odes)$
```

1. Y' = AY + F, Y(0)=Y0

```
(%i2) A:matrix([2,-4],[2,-2]);
```

(%o2) $\begin{bmatrix} 2 & -4 \\ 2 & -2 \end{bmatrix}$

```
(%i3) F:transpose([4*%e^(-2*t),0]);
```

(%o3) $\begin{bmatrix} 4\,\%e^{-2t} \\ 0 \end{bmatrix}$

```
(%i4) Y0:transpose([0,0]);
```

(%o4) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

```
(%i5) sol:odeM_ic(A,F,t,0,Y0);
```

(%o5) $\begin{bmatrix} 2\sin(2t) \\ \sin(2t)-\cos(2t)+\%e^{-2t} \end{bmatrix}$

Test:

```
(%i6) diff(sol,t)-A.sol-F$ expand(%);
```

(%o7) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

```
(%i8) ev(sol,t=0);
```

(%o8) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

2. Y' = AY, Y(0)=transpose([15,35,55,75]).

(%i9) A:matrix([4,1,1,7],[1,4,10,1],[1,10,4,1],[7,1,1,4]);

(%o9) $\begin{bmatrix} 4 & 1 & 1 & 7 \\ 1 & 4 & 10 & 1 \\ 1 & 10 & 4 & 1 \\ 7 & 1 & 1 & 4 \end{bmatrix}$

(%i10) F:transpose([0,0,0,0])$

(%i11) Y0:transpose([15,35,55,75]);

(%o11) $\begin{bmatrix} 15 \\ 35 \\ 55 \\ 75 \end{bmatrix}$

(%i12) charpoly(A, x),factor;

(%o12) $(x-15)(x-10)(x+3)(x+6)$

(%i13) sol:odeM_ic(A,F,t,0,Y0);

(%o13) $\begin{bmatrix} 27\,\%e^{15\,t}+18\,\%e^{10\,t}-30\,\%e^{-3\,t} \\ 54\,\%e^{15\,t}-9\,\%e^{10\,t}-10\,\%e^{-6\,t} \\ 54\,\%e^{15\,t}-9\,\%e^{10\,t}+10\,\%e^{-6\,t} \\ 27\,\%e^{15\,t}+18\,\%e^{10\,t}+30\,\%e^{-3\,t} \end{bmatrix}$

Test:

(%i14) diff(sol,t)-A.sol$
expand(%);

(%o15) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

(%i16) ev(sol,t=0);

(%o16) $\begin{bmatrix} 15 \\ 35 \\ 55 \\ 75 \end{bmatrix}$

matrix_exp

Function: matrix_exp(A,t)
Returns matrix exponential e^(At)
computed via Laplace transforms.

```
(%i1) matrix_exp(A,r):=
       block([n,B,s,t,Lap,f],
       n:length(A),
       B:invert(s*ident(n)-A),
       Lap(f):=ilt(f, s, t),
       matrixmap(Lap,B),
       subst(t=r,%%))$
```

Examples:

1.

```
(%i2) A:matrix([1,1],[0,1]);
```

(%o2) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

```
(%i3) matrix_exp(A,t);
```

(%o3) $\begin{bmatrix} \%e^{t} & t\,\%e^{t} \\ 0 & \%e^{t} \end{bmatrix}$

```
(%i4) e^'A=matrix_exp(A,1);
```

(%o4) $e^{A}=\begin{bmatrix} \%e & \%e \\ 0 & \%e \end{bmatrix}$

2.

```
(%i5) A:matrix([21,17,6],[-5,-1,-6],[4,4,16]);
```

(%o5) $\begin{bmatrix} 21 & 17 & 6 \\ -5 & -1 & -6 \\ 4 & 4 & 16 \end{bmatrix}$

```
(%i6) e^'A=matrix_exp(A,1);
```

(%o6) $e^{A}=\begin{bmatrix} \frac{13\,\%e^{16}}{4}-\frac{\%e^{4}}{4} & \frac{13\,\%e^{16}}{4}-\frac{5\,\%e^{4}}{4} & \frac{\%e^{16}}{2}-\frac{\%e^{4}}{2} \\ \frac{\%e^{4}}{4}-\frac{9\,\%e^{16}}{4} & \frac{5\,\%e^{4}}{4}-\frac{9\,\%e^{16}}{4} & \frac{\%e^{4}}{2}-\frac{\%e^{16}}{2} \\ 4\,\%e^{16} & 4\,\%e^{16} & \%e^{16} \end{bmatrix}$

odelinsys

Function: odelinsys(A, F, x, x0, Y0)
Find solutions of initial problem for linear system of ODEs
in matrix form: Y' = AY + F, Y(x0) = Y0.

```
(%i1) load(odes)$ load(diag)$
```

Examples:

1. Solve Y' = AY + F, Y(0) = Y0

```
(%i3) A:matrix([1,3],[-1,5]);
```

(%o3) $\begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix}$

```
(%i4) F:transpose([-x,2*x])$ Y0:transpose([3,1])$
```

```
(%i6) sol:odelinsys(A,F,x,0,Y0);
```

(%o6) $\begin{bmatrix} \frac{7\,\%e^{4x}}{32}+\frac{15\,\%e^{2x}}{8}+\frac{11x}{8}+\frac{29}{32} \\ \frac{7\,\%e^{4x}}{32}+\frac{5\,\%e^{2x}}{8}-\frac{x}{8}+\frac{5}{32} \end{bmatrix}$

Test:

```
(%i7) diff(sol,x)-A.sol-F,expand;
```

(%o7) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

2. Solve Y' = AY

```
(%i8) A:matrix([4,-1,0],[3,1,-1],[1,0,1]);
```

(%o8) $\begin{bmatrix} 4 & -1 & 0 \\ 3 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$

```
(%i9) sol:odelinsys(A,[0,0,0],t,0,[C1,C2,C3]),factor;
```

(%o9) $\begin{bmatrix} \frac{\%e^{2t}\left(t^2\,C3-t^2\,C2-2\,t\,C2+t^2\,C1+4\,t\,C1+2\,C1\right)}{2} \\ \%e^{2t}\left(t^2\,C3-t\,C3-t^2\,C2-t\,C2+C2+t^2\,C1+3\,t\,C1\right) \\ \frac{\%e^{2t}\left(t^2\,C3-2\,t\,C3+2\,C3-t^2\,C2+t^2\,C1+2\,t\,C1\right)}{2} \end{bmatrix}$

wronskian

Function: wronskian ([f_1, ..., f_n], x)
Returns the Wronskian matrix of the list of expressions [f_1, ..., f_n] in the variable x.

```
(%i1) load(odes)$
```

Examples:

1.

```
(%i2) wronskian([f(x),g(x),h(x)],x);
```

(%o2) $$\begin{bmatrix} f(x) & g(x) & h(x) \\ \frac{d}{d\,x}f(x) & \frac{d}{d\,x}g(x) & \frac{d}{d\,x}h(x) \\ \frac{d^2}{d\,x^2}f(x) & \frac{d^2}{d\,x^2}g(x) & \frac{d^2}{d\,x^2}h(x) \end{bmatrix}$$

2. Form a linear homogeneous differential equation,
knowing its fundamental system of solutions:
y1=x, y2=x^3.

```
(%i3) depends(y,x);
```

(%o3) $[y(x)]$

```
(%i4) wronskian([x,x^3,y],x);
```

(%o4) $$\begin{bmatrix} x & x^3 & y \\ 1 & 3\,x^2 & \frac{d}{d\,x}y \\ 0 & 6\,x & \frac{d^2}{d\,x^2}y \end{bmatrix}$$

```
(%i5) determinant(%)=0;
```

(%o5) $$x\left(3\,x^2\left(\frac{d^2}{d\,x^2}y\right)-6\,x\left(\frac{d}{d\,x}y\right)\right)-x^3\left(\frac{d^2}{d\,x^2}y\right)+6\,x\,y=0$$

```
(%i6) eq:expand(%/x/2);
```

(%o6) $$x^2\left(\frac{d^2}{d\,x^2}y\right)-3\,x\left(\frac{d}{d\,x}y\right)+3\,y=0$$

```
(%i7) ode2(eq,y,x);
```

(%o7) $y=\%k1\,x^3+\%k2\,x$

References:

1. http://maxima.sourceforge.net/

2. Kamke, E., 1944. Differentialgleichungen. Losungsmethoden und Losungen. AkademischeVerlagsgesellschaft, Leipzig.

3.