

Ordinary Differential Equations

An ordinary differential equation (or ODE) is a relation that contains functions of only **one independent variable**, and one or more of its **derivatives** with respect to that variable.

Example

A simple example of an ODE is Newton's second law of motion $F = ma$, which can be written as

$$F(x(t)) = m \frac{d^2 x(t)}{dt^2}$$

Where m is the independent variable, and $\frac{d^2 x(t)}{dt^2}$ is the derivative. ¹

In plain english, ODEs are equations to be solved in which the unknown element is a function, rather than a number, and in which the known information relates that function to its derivatives. ²

Application

Ordinary differential equations arise in many different contexts including geometry, mechanics, astronomy and population modelling. Many famous mathematicians have studied differential equations and contributed to the field, including Newton, Leibniz, the Bernoulli family, Riccati, Clairaut, d'Alembert and Euler.

Much study has been devoted to the solution of ordinary differential equations. In the case where the equation is linear, it can be solved by analytical methods. Unfortunately, most of the interesting differential equations are non-linear and, with a few exceptions, cannot be solved exactly. Approximate solutions are arrived at using numerical ordinary differential equations, but that's another story.

Terminology

First Order ODE

¹Remember that the distance if differentiated twice, is acceleration. $a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$

²If you still don't understand what ODEs are, don't worry, you will get it as soon as you start solving some ODEs.