Algebraic Fractions

First a bit of terminology. **Polynomials** are sums of powers of x like

$$x^3 + 3x^2 + 2x + 1$$

or

$$x^2 + 2x + 3$$

Algebraic fractions are the rations of two polynomials like

$$\frac{x^3 + 3x^2 + 2x + 1}{x^2 + 2x + 3}$$

If the numerator (top) has a higher power of x than the denominator (bottom) then the fraction is **improper**, otherwise it is **proper**.

We often need to deal with fractional expressions when we are trying to use formulae, rearranging them to extract the particular variable we are interested in. Lets's look at some examples of doing this.

Examples

Example 1 Given that $x = \frac{3}{m}$ and $y = 6m^2$, express y in terms of x.

Solution

$$y = 6(\frac{3}{x})^2 = \frac{6 \times 9}{x^2} = \frac{54}{x^2}$$

Example 2 Solve the equation

$$\frac{2x}{3} + 6 = \frac{4}{5}(x + 2)$$

Solution

We first remove the fractions by multiplying through by $3 \times 5 = 15$. Then

$$10x + 90 = 12(x + 2) = 12x + 24$$

Thus $2x = 66$ so that $x = 33$.

Example 3 Solve the equation

$$\frac{2}{x - 6} + \frac{4}{x + 3} = 1$$

Solution

We first remove the fractions by multiplying through by $(x - 6)(x + 3)$. Then

$$\frac{2(x - 6)(x + 3)}{x - 6} + \frac{4(x - 6)(x - 3)}{x + 3} = (x - 6)(x + 3)$$

so that $2x + 6 + 4x - 24 = x^2 - 3x - 18$.

Thus $x^2 - 9x = 0$ so that $x(x - 9) = 0$ and $x = 0$ or $x = 9$.