Partial Fractions

Often we are interested in either splitting up fractions or sticking them together when we are dealing with formulas. For instance the expression

$$\frac{2}{x-1} + \frac{5}{x+4}$$

can be written as a single fraction by introducing the common denominator $(x-1)(x+4)$:

$$\frac{2}{x-1} + \frac{5}{x+4} = \frac{2(x+4)+5(x-1)}{(x-1)(x+4)} = \frac{7x+3}{(x-1)(x+4)}$$

The method of **partial fractions** provides a means of performing the reverse operation.

First we describe the simplest case when

1. The denominator of the fraction can be expressed as a product of two linear factors; for example $(x-1)(x+4)$.
2. The numerator is linear; for example $7x+3$.

Examples

Example 1 Express

$$\frac{7x+3}{(x-1)(x+4)}$$

in terms of partial fractions.

Solution

Let

$$\frac{7x+3}{(x-1)(x+4)} = \frac{A}{x-1} + \frac{B}{x+4}$$

We have to find the values of A and B so the above is true for all x.

Multiplying by $(x-1)(x+4)$:

$$\frac{(7x+3)(x-1)(x+4)}{(x-1)(x+4)} = \frac{A(x-1)(x+4)}{x-1} + \frac{B(x-1)(x+4)}{x+4}$$

Hence

$$7x+3 = A(x+4) + B(x-1)$$

Now we can choose values of x to make the term involving A or the term involving B disappear.

First we will try and make the B term disappear

Put $x=1$ then $7 \times 1 + 3 = A(1+4) + 0$.

Thus $10 = 5A$ so $A = 2$.

Put $x = -4$ then $7 \times (-4) + 3 = 0 + B(-4 - 1)$.

Hence -25 = -5B so B=5.

Now that we know $A = 2$ and $B = 5$, we can put it back into the equation

$$\frac{7x+3}{(x-1)(x+4)} = \frac{2}{x-1} + \frac{5}{x+4}$$

Which is correct!

Example 2 Express

$$\frac{10x+18}{(2x+3)^2}$$

in terms of partial fractions.

Solution

Let

$$\frac{10x+18}{(2x+3)^2} = \frac{A}{2x+3} + \frac{B}{(2x+3)^2}$$

and multiply both sides by $(2x+3)^2$. Comparing the top lines

$$10x + 18 = A(2x+3) + B$$

now its time to **equate the coefficients**:

lets begin with **x coefficients**.

Equating coefficients means that you look at the equation

$$10x + 18 = A(2x+3) + B$$

and say "How many x terms do I have on the **left hand side**" and then " How many x terms do I have on the **right hand side**", do this and you should get:

$$10 = 2A$$

$$5 = A$$

Now that we have equated the x coefficient, it is time for the constants. Do the same as before, but this time with **constants** we get:

$$18 = 3A + B$$

$$18 = 15 + B$$

$$3 = B$$

After equating the coefficients we have worked out $A = 5$, and $B = 3$, we can put it back into the equation

$$\frac{10x+18}{(2x+3)^2} = \frac{5}{2x+3} + \frac{3}{(2x+3)^2}$$